THE APPLICATION OF HARMONIC-BALANCE METHODOLOGY TO THE ANALYSIS OF INJECTION LOCKING

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ABSTRACT

The paper outlines a set of software tools allowing injection locking to be quantitatively treated by harmonic-balance methodology, and discusses in detail the stability portrait of a typical injection locked microwave oscillator. The simulation can accurately describe the spreading of the unlocked oscillation spectrum near the locking range edge.

INTRODUCTION

The understanding of injection locking is still widely based on classic simplified treatments such as the Kurokawa approach [1] or the descriptive function method [2]. Although these techniques do provide a good intuition of the basic phenomenon, they rely upon drastic approximations such as neglecting the effects of harmonics, and on schematic descriptions of the active devices which are usually inadequate and difficult to derive. They are thus not compatible with the modern nonlinear CAD approach, requiring full consideration of the signal spectra, sophisticated device models, and a realistic description of the entire circuit topology. In recent years, the harmonic-balance (HB) technique has gained widespread acceptance as a numerical tool for the steady-state analysis of broad classes of nonlinear microwave circuits. Recent developments of this technique have extended its capabilities to cover local [3] and global [4, 5] stability analysis, the search for bifurcations of parametrized circuits [5], and the ability to deal with quasi-periodic (multitone) electrical regimes of autonomous circuits [6,]. In this paper we show that these new capabilities allow the injection locking problem to be investi-gated in depth by HB methodology. The advantage of harmonic balance with respect to the classic approaches [1, 2] is that it is rigorous and straightforward, and completely fulfills the requirements of a general-purpose CAD environment for high-frequency applications. The advantage of HB over time-domain nonlinear simulation lies in its much higher computational speed, allowing the efficient derivation of extensive stability portraits. Producing similar results in the time domain would be a formidable task requiring the execution of computer jobs of overwhelming numerical size.

The next section presents a synthetic discussion of the fundamental software tools that are required for a detailed frequency-domain analysis of injection locking, and of their implementation in a harmonic-balance environment. The last section describes an illustrative example, showing the kind of information that can be derived by harmonic-balance analysis.

BASIC SOFTWARE TOOLS

A numerical analysis of the injection locking phenomenon in the frequency domain requires the execution (in various combinations) of the following four fundamental operations:

i) The analysis of a microwave oscillator operating in a periodic or quasi-periodic steady state, with one or two unknown fundamental frequencies.

ii) The computation of a solution path in the state space for a parametrized oscillator.

iii) The detection of bifurcations on a known solution path.

iv) The local stability analysis of a periodic steady state.

A number of recent developments of the basic harmonic-balance technique have led to the efficient implementation of all the above capabilities in a homogeneous set of software tools. In this section we present a quick review of the related methodologies.

i) An injection-locked oscillator is a self-oscillating circuit forced by an external sinusoidal source. From the physical viewpoint, the system response to this excitation can exhibit two qualitative portraits. If the power/frequency combination of the forcing signal is such that stable locking takes place, the response is a time-periodic regime with a fundamental equal to the injected frequency, and can be found by a conventional forced circuit analysis under single-tone excitation. Otherwise, the response is quasi-periodic, with one unknown fundamental frequency, since the free oscillation is perturbed by the injection of the forcing signal. In this case the circuit can be solved by a "mixed-mode" Newton iteration [6], A phase reference is arbitrarily established for the free (unknown) fundamental by setting to zero the imaginary part of a suitable harmonic. Then the system is solved with a hybrid vector of unknowns containing the reduced set of state-variable (SV) harmonics and the unknown fundamental. In certain special circumstances, the same kind of analysis has to be carried out with both fundamental frequencies treated as unknowns (e.g., in the vicinity of a turning point of the solution path for the quasi-periodic solution), in order to avoid the singularity of the Jacobian (see point *ii)* below). From the numerical viewpoint, it is worth noting that a harmonic-balance analysis is always carried out with a predefined spectrum. If this spectrum only contains harmonics of the injected frequency, the numerical result will be a time-periodic regime even outside the stable locking range. Of course, this "locked" regime is unstable and cannot exist in reality, but this can only be established by the stability analysis tools discussed at points *iii)* and *iv)* below.

ii) In principle, the derivation of a solution path for a parametrized oscillator is only a matter of repeatedly applying the analysis algorithm. In this respect the Newton iteration represents an ideal choice, since a starting point relatively close to the solution is always available, so that its high rate of convergence in the vicinity of the solution can be fully exploited. For the first point of the curve a systematic startup procedure is available [7]. The only aspect requiring additional consideration is the occurrence of turning points in the solution path, such as points T_1, T_2 in fig. 2. At a turning point the Jacobian is singular [8], so that the Newton-iteration based HB analysis in its conventional formulation is unable to reach it [9]. In our program, this difficulty has been overcome by a novel

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implementation of the "switching-parameter" concept originally devised by Chua and Ushida for finding multiple solutions of resistive circuits [9]. In the study of injection locking, the free parameter is usually chosen as the injected frequency, as in figs. 1 - 3. Along a branch of the solution path encompassing a turning point, the roles of the reference harmonic (in this case, the fundamental of the drain voltage) and of the injected frequency are interchanged. This means that the former is treated as the independent parameter, and the latter as an unknown to be determined by the HB analysis. The switching criterion is based on the condition number of the conventional Jacobian, which is evaluated at each point of the curve after completing the HB analysis. The switching is automatically activated when the condition number raises above a prescribed threshold (10^4 in the present implementation), and is suppressed when it drops below threshold. When a turning point occurs in the solution path of a quasi-periodic regime (e.g., points T_5, T_6 in fig. 3), in the switched range the HB analysis must determine two unknown fundamental frequencies.

iii) In our implementation of the piecewise harmonic-balance technique, the nonlinear subnetwork is described by the generalized parametric equations [5]

$$\mathbf{v}(t) = \mathbf{u}\left[\mathbf{x}(t)\ ,\ \frac{d\mathbf{x}}{dt}\ ,\ \ldots,\ \frac{d^n\mathbf{x}}{dt^n}\ ,\ \mathbf{x}_D(t)\right]$$
$$\mathbf{i}(t) = \mathbf{w}\left[\mathbf{x}(t)\ ,\ \frac{d\mathbf{x}}{dt}\ ,\ \ldots,\ \frac{d^n\mathbf{x}}{dt^n}\ ,\ \mathbf{x}_D(t)\right] \qquad (1)$$

where $\mathbf{v}(t)$, $\mathbf{i}(t)$ are vectors of voltages and currents at the common ports, $\mathbf{x}(t)$ is a vector of state variables, and $\mathbf{x}_D(t)$ a vector of time-delayed state variables, i.e., $x_{Di}(t) = x_i(t - \tau_i)$. The advantages of this kind of representation are discussed elsewhere [5].

Let us now assume that a periodic or quasi-periodic solution of the circuit equations is perturbed by a superimposed sinusoidal signal of angular frequency ω. If the perturbation is small enough, it can be studied by linearizing the nonlinear subnetwork equations (1) in the neighborhood of the unperturbed steady state. This implies that the perturbed steady state may be represented as a quasi-periodic regime containing only intermodulation (IM) products of first order with respect to the perturbation (*sidebands*). A generic signal supported by the circuit thus takes the form

$$a(t) = a_{ss}(t) + \sum_{\mathbf{k}} \Delta A_{\mathbf{k}} \exp\left[j(\omega + \Omega_{\mathbf{k}})t\right] \qquad (2)$$

where $a_{ss}(t)$ is the unperturbed steady state, and the Ω_k are the spectral lines of such steady state. Due to the linearization, the phasors of the voltage and current harmonics at the sidebands are linearly related by the conversion equations of the nonlinear subnetwork. If the nonlinear devices are described by the parametric equations (1), the conversion equations are also expressed in parametric form as follows [5]

$$\Delta\mathbf{V} = \mathbf{P}\,\Delta\mathbf{X}$$
$$\Delta\mathbf{I} = \mathbf{Q}\,\Delta\mathbf{X} \qquad (3)$$

Specifically, eqn. (3) are linear maps between the spectra of the perturbations on voltages ($\Delta\mathbf{V}$), currents ($\Delta\mathbf{I}$), and state variables ($\Delta\mathbf{X}$). The corresponding linear operators $\mathbf{P}, \mathbf{Q}$ are the conversion matrices of the nonlinear subnetwork.

Now let the nonlinear circuit be parametrized by an arbitrary physical quantity p on which the circuit equations are continuously dependent. Typical choices of p for injection locking analysis are the frequency or the available power of the locking signal. Let $\mathbf{X}$ be the state vector, i.e., the set of real and imaginary parts of the state-variable harmonics. The parameter-dependent solution $\mathbf{X}(p)$ of the HB equations defines the solution path in the state space. A point $\mathbf{X}(p_B)$ of the solution path is a bifurcation if the real part of one (or more) natural frequencies of the steady state changes sign at $p = p_B$ [8]. An equation for the bifurcations of a given solution path can easily be formulated in the frequency domain. Let $\mathbf{S_L}(j\omega)$ be a block-diagonal matrix whose diagonal blocks consist of the scattering matrix of the linear subnetwork evaluated at the sidebands. Then at a bifurcation we have [4]

$$\Delta[j\omega_B, \mathbf{X}(p_B), p_B] = \det\left[(\mathbf{1} - \mathbf{S_L})\mathbf{P} - (\mathbf{1} + \mathbf{S_L})\mathbf{Q}\right] = 0 \qquad (4)$$

where p_B is the bifurcation value of the parameter, and $j\omega_B$ is the natural frequency at the bifurcation (which, by definition, is purely imaginary).

Two kinds of bifurcations are primarily of interest in the analysis of injection locking, namely, the Hopf bifurcation and the regular turning point. At a Hopf bifurcation the real parts of two complex conjugate natural frequencies change sign, so that a free oscillation starts up (or dies out) when the bifurcation is crossed along the solution path. From the conceptual viewpoint, a Hopf bifurcation can be found on a known solution path $\mathbf{X}(p)$ by solving (4) as a system of two real equations in two real unknowns ω_B, p_B. From the numerical viewpoint, the computation of the left-hand side of (4) with given values of ω_B, p_B is carried out as follows. The block scattering matrix $\mathbf{S_L}(j\omega_B)$ is obtained from a linear subnetwork analysis. The state vector $\mathbf{X}(p_B)$ is found by solving the HB equations in the way discussed at point *i)* above. The associated conversion matrices $\mathbf{P}, \mathbf{Q}$ are then computed by the general algorithm discussed in [5].

A regular turning point is a regular point of the solution path where the parameter reaches a relative maximum or minimum [8]. At a turning point one real natural frequency of the steady state changes sign, which implies that the Jacobian matrix $\mathbf{J}$ of the harmonic-balance system has a zero eigenvalue, and is thus singular [8]. Thus the exact location of a turning point may be found on a known solution path $\mathbf{X}(p)$ by solving the real equation

$$\det\left\{\mathbf{J}[\mathbf{X}(p_B), p_B]\right\} = 0 \qquad (5)$$

for the real unknown p_B. Note that good starting values are usually available for the solutions of both (4) and (5), since the approximate locations of the bifurcations can be obtained by inspection of the solution path.

iv) The exchanges of stability among the branches of the solution path originating from a bifurcation are defined by the mathematical theory of bifurcations [10]. Thus in principle, after all bifurcations have been located, the entire stability portrait can be defined by the local stability analysis of one arbitrarily selected point of the solution path. If this point is chosen as a periodic steady state, the stability analysis can be performed by numerically computing a Nyquist stability plot [3], i.e., by plotting on the complex plane the left-hand side of (4) (with ω_B, p_B replaced by ω, p) as a function of ω. In this case Δ is a periodic function of ω whose period is equal to the fundamental Ω of the steady state [3]. Thus the entire Nyquist diagram is obtained by cosidering only the finite range $0 \leq \omega \leq \Omega$. If the selected point is not a bifurcation, the origin of the complex plane does not belong to the plot because (4) is not satisfied, so that the number of encirclements of the origin is directly related to the number of unstable natural frequencies [3]. In practical applications, several local stability analyses are carried out as a matter of course, in order to double-check the accuracy of the global stability picture.

INJECTION LOCKING ANALYSIS VIA HARMONIC BALANCE

A microwave oscillator topology suitable for injection locking is the two-port configuration discussed in ref. [11]. Our purpose here is not to achieve a specific design, but rather to discuss the application of HB methodology to a typical system. We thus simply consider a circuit of this kind designed as a free-running oscillator by numerical optimization [12]. The design goal is an output power of +14 dBm at $\omega_0 = 2\pi \cdot 6$ GHz, and no attempt is made to optimize the locking bandwidth. The use of stub matching networks results in an effective Q [1] of about 15.

Now let the oscillator be forced by a sinusoidal signal of available power P_{in} and angular frequency ω_1. Figs. 1, 2, and 3 show the stability portraits obtained when the locking frequency is used as the independent parameter at three different power levels of the injected signal.

A stability portrait of the kind shown in fig. 1 is obtained for relatively high values of injected power (approximately $P_{in} >$ +1.8 dBm for the circuit under consideration). Fig. 1 is drawn for P_{in} = +3 dBm. The figure shows the two-dimensional projection of the solution path of the harmonic-balance equations in the parameter range 5.5 GHz $\leq \omega_1/2\pi \leq$ 6.5 GHz. The branch AH_1H_2B represents the time-periodic solution of fundamental frequency ω_1, i.e., the locked regime. Any state of this branch lying between H_1 and H_2 can be shown to be stable by a local stability analysis based on the Nyquist method (point *iv)* of the previous section). In the HB analysis of the locked regime, four harmonics of the input frequency (including the fundamental) are taken into account. For the sake of graphical representation, the output power at the locking frequency is used as a quantity representative of the system state along the branch AH_1H_2B. Two Hopf bifurcations H_1, H_2, are encountered on this branch. H_1 is supercritical [8] in the decreasing parameter sense, while H_2 is supercritical in the increasing parameter sense. Thus, according to the results of bifurcation theory [8, 10] each of the states belonging to the branches AH_1 and H_2B has two complex conjugate natural frequencies with positive real parts, and is therefore unstable. Stable locking thus takes place only along the branch H_1H_2, so that the *stable locking range* is given by the frequency distance between the two Hopf bifurcations. Below H_1 and above H_2 the solution path has two branches: the already mentioned branch representing an unstable locked regime (AH_1 and H_2B, respectively), and a further branch representing a stable quasi-periodic regime. On this branch the oscillator is unlocked, so that a free oscillation with a fundamental frequency $\omega_2 \neq \omega_0$ coexists and intermodulates with the injected signal. This branch is represented in fig. 1 by two curves, which are generated by two different projections (orthogonal and parallel to the hyperplane of the harmonics of ω_1) of the branch itself on the figure plane. Thus the couples of points C/C', H_1/H_1', H_2/H_2', D/D' are the images of the same four points of the solution path created by the two projections (note that H_1', H_2' fall outside the power range spanned by the ordinate of fig. 1). On the branches CH_1 and H_2D (obtained from the orthogonal projection), the quantity used as representative of the system state is still the output power at ω_1. On the branches $C'H_1'$ and $H_2'D'$ (obtained from the parallel projection), it is the output power at ω_2.

In an intermediate range of input power values (approximately -2.21 dBm < P_{in} < +1.8 dBm for the circuit under consideration), the projection of the solution path takes the shape given in fig. 2. Fig. 2 is drawn for P_{in} = -1 dBm in the parameter range 5.8 GHz $\leq \omega_1/2\pi \leq$ 6.2 GHz. One evident difference with respect to fig. 1 is the occurrence of turning points in the solution path. Every state belonging to the branches T_1T_2 and T_3T_4 has one positive real natural frequency and is thus unstable [8]. The Hopf bifurcations lie between the turning points T_1, T_2 and T_3, T_4, respectively. The states belonging to H_1T_2 and T_3H_2 are therefore unstable, so that the bounds of the stable locking range are now represented by the turning points T_2, T_3. The oscillator exhibits two small hysteresis cycles at the locking band edges.

For a critical value of the input power ($P_{in} \cong$ -2.21 dBm for the circuit under consideration), the turning points T_1, T_4 merge, and the solution path exhibits a double point. Below this value, the projection of the solution path takes the shape given in fig. 3 (the figure is actually drawn for P_{in} = -3 dBm). The locked regimes are now represented by two disjoint branches AB and OH_1H_2O. All the periodic states belonging to the branch AB have two complex conjugate natural frequencies with positive real parts and are thus unstable. The Hopf bifurcations H_1, H_2 now lie between the turning points T_2, T_3. Each state belonging to the branches T_2H_1 and H_2T_3 is thus unstable because of a couple of complex conjugate natural frequencies having positive real parts. Stable locking therefore takes place only on the branch H_1H_2. H_1 is a subcritical Hopf bifurcation [8] in the decreasing parameter sense, while H_2 is subcritical in the increasing parameter sense. Thus the quasi-periodic bifurcated branches are unstable in the vicinity of the Hopf bifurcations because of one positive real natural frequency. These natural frequencies change sign at the turning points T_5, T_6, so that the quasi-periodic branches $CT_5/C'T_5'$ and $T_6D/T_6'D'$ are stable. The oscillator exhibits two hysteresis cycles near the locking band edges.

In the HB analysis of the quasi-periodic steady states far enough from the Hopf bifurcations, all IM products of the two fundamentals up to the fourth order are taken into account. However, as the locking band edges are approached, the spectrum quickly spreads out, and at the same time the fundamental ω_2 of the free-running oscillation approaches the locking frequency ω_1. Thus the number of spectral lines to be considered in the harmonic-balance simulation becomes larger and larger. An example of this behavior is given in fig. 4, where the near-carrier portion of the load power spectrum is plotted for a quasi-periodic regime corresponding to point E/E' in fig. 3. This spectrum is produced by an HB analysis accounting for IM products up to the 13th order. The injected frequency is 5.94499 GHz, and the free-running fundamental is found to be 5.97469 GHz. Fig. 4 exibits the typical near-triangular spectrum of the unlocked oscillation, with most of the spectral lines crowded on the ω_0 side of the injected frequency.

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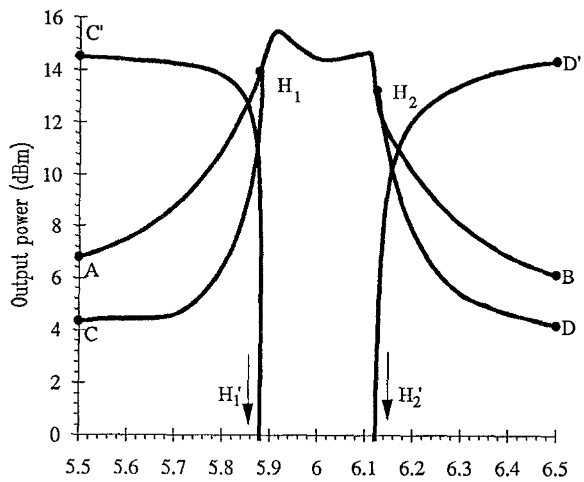

Fig. 1 - Stability portrait of a forced oscillator at a high injected power level.

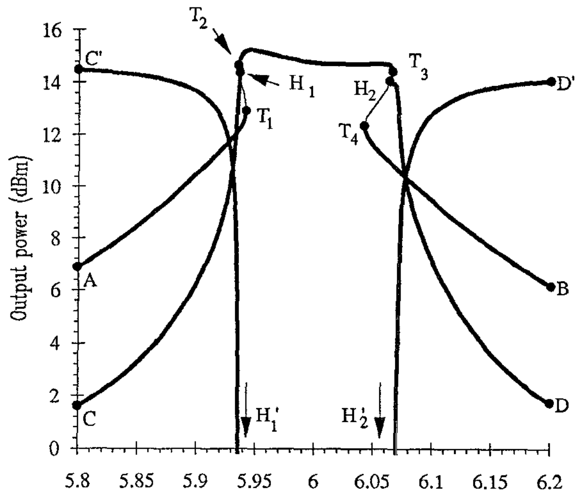

Fig. 2 - Stability portrait of a forced oscillator at an intermediate injected power level.

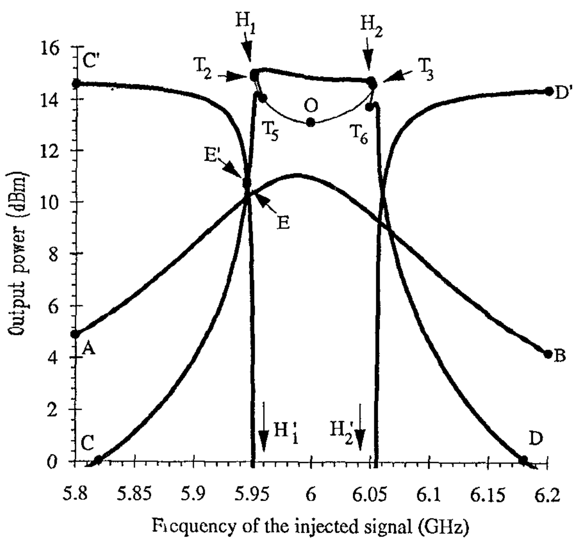

Fig. 3 - Stability portrait of a forced oscillator at a low injected power level.

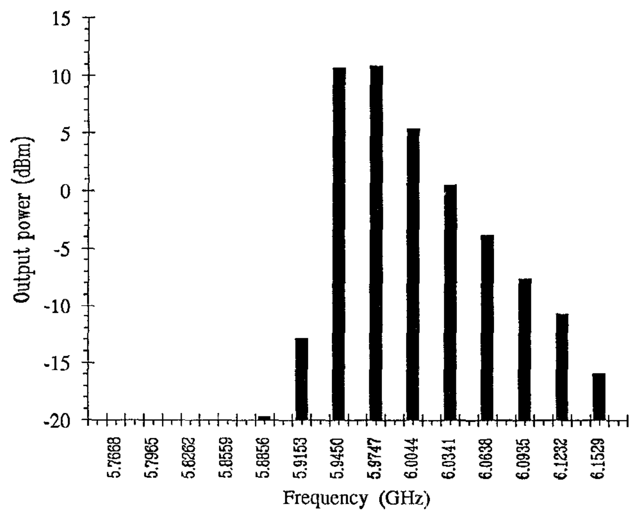

Fig. 4 - Near-carrier spectrum of a forced oscillator outside the locking range.